

Noether Charge and Black Hole Entropy in Modified Theories of Gravity

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Abstract

The entropy of black holes in modified theories of gravity is examined in the Palatini formalism using the Noether Charge approach. It is shown that, if the gravitational coupling constant is properly identified, the entropy of a black hole is one-quarter of the horizon area in $f(R)$ theories coupled to conformally invariant matter. If matter is present that is not conformally invariant the entropy is still proportional to the area of the black hole, but the coefficient is generally not one-quarter. The entropy of black holes in generalized dilaton theories and in theories with Lagrangians that depend on an arbitrary function of the Ricci tensor are also examined.

1 Introduction

Over the last few years there has been a significant amount of interest in modifications of gravity as an explanation for the observed accelerated expansion of the Universe [1] (see also [2]). There are two approaches to derive the field equations from the action in these theories, the metric approach and the Palatini approach. In the metric approach the connection is taken to be the Christoffel symbol and the action is varied with respect to the metric. In the Palatini approach the metric and connection are varied independently. For the action of general relativity these approaches produce the same field equations. However, for other actions they produce different theories.

In this paper I examine the entropy of black holes in modified theories of gravity in the Palatini formalism using the Noether charge approach as developed by Wald [3] (see also [4, 5, 6, 7]). In this approach to black hole entropy the first law of black hole mechanics is written as

$$\delta \int_{\Sigma} \mathbf{Q} = \delta E - \Omega_H^{(A)} \delta J_{(A)} , \quad (1)$$

where \mathbf{Q} is the Noether potential associated with diffeomorphisms on the manifold, Σ is the bifurcation surface of the black hole, E is the canonical energy, $J_{(A)}$ is the canonical angular momentum and $\Omega_{(A)}$ is the angular velocity of the horizon. If \mathbf{Q} can be written in terms of local geometric quantities and the matter fields present in the spacetime the entropy will be given by

$$S_{BH} = \frac{2\pi}{\kappa} \int_{\Sigma} \mathbf{Q} , \quad (2)$$

where κ is the surface gravity of the black hole. In fact, it has been shown [6] that the above integral can be taken over any cross-section of the Killing horizon.

In section 2 the Noether charge approach is applied to gravitational theories in the Palatini formalism. In section 3 the entropy of black holes in $f(R)$ theories of gravity is investigated. In section 4 theories with Lagrangians that depend on arbitrary functions of the Ricci tensor and generalized dilaton theories are examined.

2 Noether Charge and Black Hole Entropy

In this section I will apply the Noether charge approach, developed by Wald [3] in the metric formalism, to calculate black hole entropy in the Palatini formalism (see ([8] for work on black hole entropy in the metric and other approaches). The Lagrangian D-form in D dimensions will be taken to be

$$\mathbf{L} = \frac{1}{16\pi G} f(g^{\mu\nu}, R^{\alpha}_{\mu\nu\lambda}, \psi, \dots) \epsilon , \quad (3)$$

where ϵ is the volume form on the manifold,

$$R^{\alpha}_{\mu\nu\lambda} = \partial_{\nu} \Gamma^{\alpha}_{\mu\lambda} - \partial_{\lambda} \Gamma^{\alpha}_{\mu\nu} + \Gamma^{\alpha}_{\nu\sigma} \Gamma^{\sigma}_{\mu\lambda} - \Gamma^{\alpha}_{\lambda\sigma} \Gamma^{\sigma}_{\mu\nu} , \quad (4)$$

ψ denotes the matter fields and ... denotes derivatives of $g^{\mu\nu}$ and $R^\alpha_{\mu\nu\lambda}$. Note that $g^{\mu\nu}$ and $\Gamma^\alpha_{\mu\nu}$ are to be taken as independent field variables and that $\Gamma^\alpha_{\mu\nu}$ is taken to be symmetric in μ and ν . Let $\phi = (g^{\mu\nu}, \Gamma^\alpha_{\mu\nu}, \psi)$ denote the field variables. The variation in the Lagrangian associated with the variation in the field variables, $\delta\phi$, is given by

$$\delta\mathbf{L} = \mathbf{E} \cdot \delta\phi + \mathbf{d}\Theta(\phi, \delta\phi) , \quad (5)$$

where $\mathbf{E} \cdot \delta\phi$ implies a summation over field and spacetime indices. The Noether current, \mathbf{J} , associated with the diffeomorphisms generated by the vector field ξ^μ is defined to be

$$\mathbf{J} = \Theta(\phi, L_\xi\phi) - \mathbf{L}\xi^\mu , \quad (6)$$

where $L_\xi\phi$ is the Lie derivative of ϕ with respect to ξ^μ . At this point one might be concerned by the fact that $\Gamma^\alpha_{\mu\nu}$ is not a tensor (although $\delta\Gamma^\alpha_{\mu\nu}$ is a tensor). However, defining

$$(L_\xi\Gamma)^\alpha_{\mu\nu}(x) = \lim_{\lambda \rightarrow 0} \left[\frac{\bar{\Gamma}^\alpha_{\mu\nu}(x) - \Gamma^\alpha_{\mu\nu}(x)}{\lambda} \right] \quad (7)$$

with $\bar{x}^\mu = x^\mu - \lambda\xi^\mu$ gives the covariant expression

$$(L_\xi\Gamma)^\alpha_{\mu\nu} = \nabla_\mu \nabla_\nu \xi^\alpha - R^\alpha_{\nu\mu\rho} \xi^\rho . \quad (8)$$

When the field equations are satisfied (i.e. when $\mathbf{E} = 0$) it can be shown that $\mathbf{dJ} = 0$ for all ξ^μ . Thus, in this case, there exists a Noether potential, \mathbf{Q} , which satisfies $\mathbf{J} = \mathbf{dQ}$ [9]. The Noether potential \mathbf{Q} is constructed out of the field variables ϕ , their variations $\delta\phi$, and the vector field ξ^μ .

Now consider a black hole spacetime with a bifurcation (D-2) surface Σ and a Killing vector

$$\xi^\mu = t^\mu + \Omega_H^{(A)} \phi_{(A)}^\mu \quad (9)$$

which vanishes on Σ . The vector t^μ is the stationary Killing vector with unit norm at infinity, $\psi_{(A)}^\mu$ are axial Killing vectors, and $\Omega_H^{(A)}$ is the angular velocity of the horizon. One should note that ξ_μ is defined with respect to the connection, $\tilde{\nabla}_\mu$, that is compatible with $g_{\mu\nu}$ (i.e. $(L_\xi g)_{\mu\nu} = \tilde{\nabla}_\mu \xi_\nu + \tilde{\nabla}_\nu \xi_\mu = 0$).

The expression

$$\delta \int_\Sigma \mathbf{Q} = \delta E - \Omega_H^{(A)} \delta J_{(A)} , \quad (10)$$

where E is the canonical energy and $J_{(A)}$ is the canonical angular momentum is also valid in the Palatini formalism. To be able to interpret the left hand side of (10) as being proportional to the entropy the dependence of \mathbf{Q} on ξ^μ must be eliminated. As in the metric formulation all second order and higher derivatives of ξ^μ can be reduced to the first order derivatives $\nabla_\mu \xi_\nu$. If ∇_μ is compatible with some metric $h_{\mu\nu}$ then $\nabla_\mu \xi_\nu = \kappa \epsilon_{\mu\nu}$ where κ is the surface gravity and $\epsilon_{\mu\nu}$ is the binormal to Σ (defined relative to $h_{\mu\nu}$). Since $\xi^\mu = 0$ on Σ the dependence of \mathbf{Q} on ξ^μ can be eliminated. Defining $\tilde{\xi}^\mu = \kappa \xi^\mu$ we have $\delta\mathbf{Q} = \kappa \delta\tilde{\mathbf{Q}}$ and the entropy of a black hole is given by

$$S = 2\pi \int_\Sigma \tilde{\mathbf{Q}} . \quad (11)$$

3 Entropy in f(R) Theories

Consider Lagrangians of the form $\mathbf{L} = L\epsilon$ in $D > 2$ with

$$L = \frac{1}{16\pi G}f(R) + L_M , \quad (12)$$

where $R_{\mu\nu} = R^\alpha_{\mu\alpha\nu}$, $R = g^{\mu\nu}R_{\mu\nu}$, and L_M is the matter Lagrangian. Varying the action with respect to $g^{\mu\nu}$ gives

$$f'(R)R_{(\mu\nu)} - \frac{1}{2}f(R)g_{\mu\nu} = 8\pi GT_{\mu\nu} , \quad (13)$$

where $f' = df/dR$ and $R_{(\mu\nu)}$ is the symmetric part of the Ricci tensor. Varying the action with respect to $\Gamma^\alpha_{\mu\nu}$ and simplifying gives

$$\nabla_\alpha [\sqrt{-g}f'g^{\mu\nu}] = 0 . \quad (14)$$

Defining $h_{\mu\nu}$ by $\sqrt{-h}h^{\mu\nu} = f'\sqrt{-g}g^{\mu\nu}$ gives

$$\nabla_\alpha [\sqrt{-h}h^{\mu\nu}] = 0 , \quad (15)$$

so that the connection is compatible with $h_{\mu\nu}$. Contracting (13) over μ and ν gives

$$Rf'(R) - \frac{D}{2}f(R) = 8\pi GT , \quad (16)$$

which allows us to write $R = R(T)$.

First consider the entropy of vacuum spacetimes. In this case (16) implies that R is a constant (unless $f \propto R^{D/2}$) and that ∇_μ is also compatible with $g_{\mu\nu}$. Using (6) and (8) gives

$$J_{\mu_2 \dots \mu_D} = \frac{1}{8\pi G} \left[\nabla_\lambda \left(f' \nabla^{[\lambda} \xi^{\alpha]} \right) + \left(f' R^\alpha{}_\lambda - \frac{1}{2} \delta^\alpha{}_\lambda f \right) \xi^\lambda \right] \epsilon_{\alpha \mu_2 \dots \mu_D} , \quad (17)$$

where indices on ∇_μ and $R_{\mu\nu}$ have been raised with $g^{\mu\nu}$. It is also important to note that ξ_μ is raised by the metric $g^{\mu\nu}$, since the Killing vector ξ^μ is used to determine the energy and momentum of the spacetime with metric $g_{\mu\nu}$. Thus, when the field equations are satisfied

$$J_{\mu_2 \dots \mu_D} = \frac{f'}{8\pi G} \nabla_\lambda \left(\nabla^{[\lambda} \xi^{\alpha]} \right) \epsilon_{\alpha \mu_2 \dots \mu_D} . \quad (18)$$

The Noether potential \mathbf{Q} can then be written as

$$Q_{\mu_3 \dots \mu_D} = -\frac{f'}{16\pi G} \left(\nabla^\alpha \xi^\beta \right) \epsilon_{\alpha \beta \mu_3 \dots \mu_D} . \quad (19)$$

This is the same expression as one obtains in general relativity except for the constant factor f' . The entropy of a black hole is therefore given by

$$S_{BH} = f' \left(\frac{A}{4G} \right) . \quad (20)$$

Now consider spacetimes with matter. Assuming that L_M is independent of the connection it is easy to see that the matter Lagrangian will contribute the same terms to (17) as it does in the metric approach. Forms of matter such as electromagnetic fields and scalar fields only contribute terms that, with the curvature terms, vanish when the field equations are satisfied. Thus, when the field equations are satisfied the Noether current is given by

$$J_{\mu_2 \dots \mu_D} = \frac{1}{8\pi G} \nabla_\lambda \left[\nabla_\sigma \left(h^{\sigma[\lambda} \xi^{\alpha]} \right) \right] \epsilon_{\alpha \mu_2 \dots \mu_D}^{(h)} , \quad (21)$$

where $\epsilon^{(h)}$ is the volume form associated with the metric $h_{\mu\nu}$. From

$$h_{\mu\nu} = (f')^p g_{\mu\nu} , \quad (22)$$

where $p = 2/(D - 2)$, the Noether charge can be written as

$$Q_{\mu_3 \dots \mu_D} = -\frac{1}{16\pi G} f' g^{\alpha\sigma} \nabla_\sigma \left[(f')^p h^{\beta\lambda} \xi_\lambda \right] \epsilon_{\alpha\beta\mu_3 \dots \mu_D} . \quad (23)$$

This can be expanded to obtain

$$Q_{\mu_3 \dots \mu_D} = -\frac{1}{16\pi G} \left[f' g^{\alpha\sigma} \left(g^{\beta\lambda} \nabla_\sigma \xi_\lambda + (f')^{-p} \nabla_\alpha [(f')^p] \xi^\beta \right) \right] \epsilon_{\alpha\beta\mu_3 \dots \mu_D} . \quad (24)$$

Since $\xi^\beta = 0$ on Σ the Noether potential \mathbf{Q} reduces to (19) on Σ , with ∇_σ compatible with $h_{\mu\nu}$ not with $g_{\mu\nu}$. However, \mathbf{Q} is independent of the connection, so that the entropy is given by (20) if f' is constant on Σ .

Before concluding that the entropy of a black hole is f' times one-quarter of its area it is important to consider the coupling of matter to the gravitational field. For simplicity consider conformally invariant matter with $T = 0$. From (16) we see that R is a constant and the connection is compatible with $g_{\mu\nu}$. The field equation (13) can be written as

$$G_{\mu\nu}(g) = 8\pi \left(\frac{G}{f'} \right) T_{\mu\nu} - \left(\frac{D-2}{2D} \right) R g_{\mu\nu} , \quad (25)$$

where $G_{\mu\nu}(g)$ denotes the Einstein tensor constructed from $g_{\mu\nu}$. The effective Newton's constant is therefore given by

$$G_N = \frac{G}{f'} \quad (26)$$

and the entropy is given by

$$S_{BH} = \frac{A}{4G_N} . \quad (27)$$

Thus, the entropy of a black hole in a spacetime with conformally invariant matter is one-quarter of its area. This will also hold in vacuum spacetimes.

Another way to look at this is to consider the “canonical” energy

$$E = \int_{\infty} (\mathbf{Q}[t] - t \cdot \mathbf{B}) \quad (28)$$

where

$$\delta \int_{\infty} \xi \cdot \mathbf{B} = \int_{\infty} \xi \cdot \boldsymbol{\Theta} . \quad (29)$$

Iyer and Wald have shown [4] that E is the ADM mass for general relativity. If G is taken to be Newton’s constant then $\mathbf{Q}_f = f' \mathbf{Q}_{GR}$ and $E_f = f' E_{GR}$, where \mathbf{Q}_f refers to \mathbf{Q} in the $f(R)$ theory and \mathbf{Q}_{GR} refers to \mathbf{Q} in general relativity. However, vacuum $f(R)$ theories are equivalent to Einstein’s theory (plus a possible cosmological constant)[10]. Thus, it makes sense to define Newton’s constant by (26), so that the energies coincide.

Now consider spacetimes containing matter that is not conformally invariant. Equation (25) holds, with additional terms on the right hand side due to the fact that ∇_{μ} is no longer compatible with $g_{\mu\nu}$. In addition f' is no longer constant. If the energy-momentum tensor of the non-conformally invariant matter vanishes at infinity f' will approach a constant f'_{∞} at large distances from the black hole. In this case Newton’s constant will be taken to be G/f'_{∞} and the entropy will be given by

$$S = \left(\frac{f'_{\Sigma}}{f'_{\infty}} \right) \frac{A}{4G_N} . \quad (30)$$

if f' is constant on Σ . Thus, the entropy of a black hole is generally not equal to one-quarter of its area (in Planck units) if non-conformally invariant matter is present.

4 Additional Examples

Consider Lagrangians of the form $\mathbf{L} = L\epsilon$ with

$$L = \frac{1}{16\pi G} \sqrt{-g} f(g^{\mu\nu}, R_{(\mu\nu)}) . \quad (31)$$

Varying the action with respect to $g_{\mu\nu}$ gives

$$f_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} = 0 , \quad (32)$$

where $f_{\mu\nu} = \partial f / \partial g^{\mu\nu}$. Varying the action with respect to $\Gamma_{\mu\nu}^{\alpha}$ gives

$$\nabla \left[\sqrt{-g} \mathfrak{S}^{\mu\nu} \right] = 0 \quad (33)$$

where $\mathfrak{S}^{\mu\nu} = \partial f / \partial R_{(\mu\nu)}$. Now define

$$\sqrt{-h} h^{\mu\nu} = \sqrt{-g} \mathfrak{S}^{\mu\nu} \quad (34)$$

and the connection is again compatible with $h_{\mu\nu}$. It is easy to see that

$$g^{\mu\alpha} f_{\alpha\nu} = \mathfrak{S}^{\mu\alpha} R_{(\alpha\nu)} \quad (35)$$

and that \mathbf{J} is given by (21) when the field equations are satisfied. It has been shown [11, 12, 13] that the field equations in this theory are equivalent to Einstein's theory plus a possible cosmological constant (except on a set of measure zero). Since $\mathfrak{S}^{\mu\nu}$ is constructed from $g^{\mu\nu}$ and $R_{(\mu\nu)}$ it must be of the form

$$\mathfrak{S}^{\mu\nu} = \lambda g^{\mu\nu} \quad (36)$$

when the field equations are satisfied, where λ is a constant which I will assume is nonzero. From (34) and (36) we have

$$\sqrt{-h} h^{\mu\nu} = \lambda \sqrt{-g} g^{\mu\nu} , \quad (37)$$

which implies that $S_{BH} = \lambda(A/4G)$. Including matter in (32) and using (35) and (36) shows that the effective gravitational coupling is $G_N = G/\lambda$. This can also be seen by equating the energy in these theories with the energy in general relativity. Thus, $S_{BH} = A/4G_N$.

As a final example consider the Lagrangian $\mathbf{L} = L\epsilon$ where

$$L = e^{-2\phi} \left[\frac{1}{16\pi G} f(R) - \frac{1}{2} \alpha \nabla_\mu \phi \nabla^\mu \phi \right] , \quad (38)$$

and α is a constant. The field equations that follow from varying the metric and connection are

$$f' R_{(\mu\nu)} - \frac{1}{2} f g_{\mu\nu} = 8\pi G \alpha \left[\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 \right] \quad (39)$$

and

$$\nabla_\alpha \left[\sqrt{-g} e^{-2\phi} f' g^{\mu\nu} \right] = 0 . \quad (40)$$

Defining $\sqrt{-h} h^{\mu\nu} = \sqrt{-g} e^{-2\phi} f' g^{\mu\nu}$ gives $\nabla_\alpha (\sqrt{-h} h^{\mu\nu}) = 0$. The Noether charge is given by

$$Q_{\mu_3 \dots \mu_D} = -\frac{1}{16\pi G} e^{-2\phi} f' g^{\alpha\sigma} (\nabla_\sigma \xi^\beta) \epsilon_{\alpha\beta\mu_3 \dots \mu_D} , \quad (41)$$

and the entropy is

$$S_{BH} = e^{-2\phi(\Sigma)} f'_\Sigma \left(\frac{A}{4G} \right) , \quad (42)$$

where $\phi^{(\Sigma)}$ and f'_Σ are evaluated on Σ (assuming that they are constant on Σ). However, from (39) one can define $G_N = G/f'_\infty$ and this gives

$$S_{BH} = e^{-2\phi(\Sigma)} \frac{f'_\Sigma}{f'_\infty} \left(\frac{A}{4G_N} \right) \quad (43)$$

for the entropy of a black hole. In string theory $\alpha \propto 1/G$, so that equation (39) is actually independent of G . One can still define $G_N = G/f'_\infty$, since the coefficient of $R_{(\mu\nu)}$ is f'/G .

5 Conclusion

In this paper I examined black hole entropy in generalized theories of gravity in the Palatini formalism using the Noether charge approach. It was shown that the entropy of a black hole in $f(R)$ theories with conformally invariant matter is given by

$$S_{BH} = \frac{A}{4G_N} , \quad (44)$$

where $G_N = G/f'$ is the effective Newtonian constant for the theory and f' is a constant. For theories with matter that is not conformally invariant f' is not a constant, $G_N = G/f'_\infty$ and

$$S = \left(\frac{f'_\Sigma}{f'_\infty} \right) \frac{A}{4G_N} , \quad (45)$$

where f'_Σ is the value of f' evaluated on the bifurcation surface and f'_∞ is its value at spatial infinity. Vacuum theories with Lagrangians depending on arbitrary functions of the symmetric part of the Ricci tensor were examined. The field equations of these theories are equivalent to Einstein's theory plus a possible cosmological constant and the entropy was shown to be one-quarter of the surface area. Finally generalized dilaton theories of the form

$$L = e^{-2\phi} \left[\frac{1}{16\pi G} f(R) - \frac{1}{2} \alpha \nabla_\mu \phi \nabla^\mu \phi \right] \quad (46)$$

were considered and the entropy of a black hole was shown to be

$$S_{BH} = e^{-2\phi(\Sigma)} \frac{f'_\Sigma}{f'_\infty} \left(\frac{A}{4G_N} \right) , \quad (47)$$

where $\phi(\Sigma)$ and f'_Σ are evaluated on Σ (assuming that they are constant on Σ) and f'_∞ is the value of f' at infinity.

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References

- [1] S.Capozziello, V.F. Cardone, S. Carloni, A. Troisi, Int. J. Mod. Phys. **D12**, 1969 (2003); S.M. Carroll, V. Duvvuri, M. Trodden, and M.S. Turner, Phys. Rev. **D70**, 043528 (2004); D.N. Vollick, Phys. Rev. **D68**, 063510 (2003)

- [2] S. Nojiri and S. D. Odintsov, Phys. Lett. B**576**, 5 (2003); A.D. Dolgov and M. Kawasaki, Phys. Lett. B**573**, 1 (2003); X.H. Meng and P. Wang, Class. Quant. Grav. **20**, 4949 (2003); S. Nojiri and S.D. Odintsov, Phys. Rev. D**68**, 123512 (2003); X.H. Meng and P. Wang, Class. Quant. Grav. **21**, 951 (2004); S. Nojiri and S.D. Odintsov, Gen. Rel. Grav. **36** 1765 (2004); X.H. Meng and P. Wang, Phys. Lett. B**584**, 1 (2004); S. Nojiri and S.D. Odintsov, Mod. Phys. Lett. A**19**, 627 (2004); X.H. Meng, P. Wang, Class. Quant. Grav. **21**, 2029 (2004); G. Allemandi, A. Borowiec, M. Francaviglia, Phys. Rev. D**70**, 043524 (2004); G.M. Kremer and D.S.M. Alves, Phys. Rev. D**70**, 023503 (2004); G. Allemandi, A. Borowiec and M. Francaviglia, Phys. Rev. D**70**, 103503 (2004); M.C.B. Abdalla, S. Nojiri and S.D. Odintsov, Class. Quant. Grav. **22**, L35 (2005); S.M. Carroll, A. De Felice, V. Duvvuri, D.A. Easson, M. Trodden and M.S. Turner Phys. Rev. D**71**, 063513 (2005); S. Capozziello, V.F. Cardone and M. Francaviglia, Gen. Rel. Grav. **38**, 711 (2006); S. Capozziello, V.F. Cardone, M. Funaro and S. Andreon, Phys. Rev. D**70**, 123501 (2004); S. Carloni, P.K.S. Dunsby and S. Capozziello and A. Troisi, Class. Quant. Grav. **22**, 4839 (2005); D.A. Easson, Int. J. Mod. Phys. A**19** 5343 (2004); S. Capozziello, V.F. Cardone and A. Troisi, Phys. Rev. D**71**, 043503 (2005); S. Das, N. Banerjee and N. Dadhich, Class. Quant. Grav. **23**, 4159 (2006); I. Brevik, O. Gorbunova and Y.A. Shaibo, Int. J. Mod. Phys. D**14** 1899 (2005); S. Capozziello, V.F. Cardone, E. Elizalde, S. Nojiri and S.D. Odintsov, Phys. Rev. D**73** 043512 (2006); T. Koivisto and H. Kurki-Suonio, Class. Quant. Grav. **23**, 2355 (2006); N.J. Poplawski, Class. Quant. Grav. **23**, 2011 (2006); O. Mena, J. Santiago and J. Weller, Phys. Rev. Lett. **96**, 041103 (2006); M. Amarzguoui, O. Elgaroy, D.F. Mota and T. Multamaki, Astron. Astrophys. **454** 707 (2006); T. Koivisto, Phys. Rev. D**73**, 083517 (2006); L. Amendola, D. Polarski and S. Tsujikawa, Phys. Rev. Lett. **98**, 131302 (2007); S. Capozziello, S. Nojiri, S.D. Odintsov and A. Troisi, Phys. Lett. B**639**, 135 (2006); S.M. Carroll, I. Sawicki, A. Silvestri and Mark Trodden, New J. Phys. **8**, 323 (2006); J. A. R. Cembranos, Phys.Rev. D**73**, 064029 (2006); S. Fay, R. Tavakol, S. Tsujikawa, Phys. Rev. D**75**, 063509 (2007); I. Sawicki and W. Hu, Phys. Rev. D**75**, 127502 (2007); S. Fay, S. Nesseris and L. Perivolaropoulos, Phys. Rev. D**76**, 063504 (2007); K. Uddin, J.E. Lidsey, R. Tavakol, Class. Quant. Grav. **24**, 3951 (2007); M.S. Movahed, S. Baghran and S. Rahvar, Phys. Rev. D**76**, 044008 (2007); T. Koivisto, Phys. Rev. D**76**, 043527 (2007); Yong-Seon Song, H. Peiris and W. Hu, Phys. Rev. D**76**, 063517 (2007);
- [3] R. M. Wald, Phys. Rev. D**48**, R3427 (1993)
- [4] V. Iyer and R.M. Wald, Phys. Rev. D**50**, 846 (1994)
- [5] M. Visser, Phys. Rev. D**48**, 5697 (1993);
- [6] T. Jacobson, G. Kang and R.C. Myers, Phys. Rev. D**49**, 6587 (1994);
- [7] V. Iyer and R.M. Wald, Phys. Rev D**52**, 4430 (1995)

- [8] F. Brisce and E. Elizade, arXiv:hep-th/0708.0432; G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov and S. Zerbini, J. Cosmol. Astropart. Phys. **0502**, 010 (2005); G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov and S. Zerbini, Phys. Rev. **D73**, 084007 (2006); M. Akbar and Rong-Gen Cai, Phys. Lett. **B648**, 243 (2007); H. Mohseni Sadjadi, arXiv:gr-qc/0709.2435; A. Mukhopadhyay and T. Padmanabhan, Phys. Rev. D **74**, 124023 (2006); A. Paranjape, S. Sarkar and T. Padmanabhan, Phys. Rev. D **74**, 104015 (2006); P. Wang, Phys. Rev. **D72**, 024030 (2005); M. Cvitan and S. Pallua, Phys. Rev. **D71**, 104032 (2005); I. Brevik, S. Nojiri, S.D. Odintsov and L. Vanzo, Phys. Rev. D **70**, 043520 (2004); J.I. Koga and K.I. Maeda, Phys. Rev. **D58**, 064020 (1998); T. Jacobson, G. Kang and R.C. Myers, Phys. Rev. **D52**, 3518 (1995)
- [9] R.M. Wald, J. Math. Phys. **31**, 2378 (1990)
- [10] M. Ferraris, M. Francaviglia and I. Volovich, Class. Quant. Grav. **11** , 1505 (1994)
- [11] V. Tapia and M. Ujevic, Class. Quant. Grav. **15**, 3719 (1998)
- [12] G. Allemandi, A. Borowiec and M. Francaviglia, Phys. Rev. **D70**, 103503 (2004)
- [13] A. Borowiec, M. Ferraris, M. Francaviglia and I. Volovich, Class. Quant. Grav. **15**, 43 (1998)